

Transition redshift from the V-reconstruction method

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We show the effectiveness of using the procedure of the potential reconstruction (the V-method) in the estimation of the transition redshift corresponding to switch from deceleration to acceleration phase of the Universe. We investigate the FRW models with dark energy by applying the particle-like description of their dynamics. In this picture the evolution of the universe is represented by motion of a fictitious particle in the one-dimensional potential $V(a)$ or $V(z)$ where a is the scale factor and z is the redshift. The V-method solves the inverse problem where we reconstruct the potential function from empirical data. We use the Riess et al.'s gold and gold+silver samples of SN Ia data in this reconstruction. In the same framework we obtain both the estimates of the form of the equation of state for dark energy and the transition redshift. We obtain that transition redshift is $0.38^{+0.10}_{-0.07}$ (gold subset) and $0.37^{+0.06}_{-0.07}$ (gold+silver sample) when the linear model of dark energy ($w(z) = w_0 + w_1 z$) and $\Omega_{m,0} = 0.3$ are assumed. We compare the estimation of transition redshift with the Riess et al. results. The V-method also allows us to find the value of Hubble function at the moment of transition.

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I. INTRODUCTION

The recent supernovae observations indicate that our universe is currently accelerating [1, 2]. However, from the standard cosmological model we know that earlier the universe decelerated because of domination of matter, and there should exist the moment when the switch from the deceleration phase to the acceleration phase takes place. The value of redshift for this transition z_T has been recently found by Riess et al. [3]. They obtained $z_T = 0.46 \pm 0.13$ using a kinematic model independent on the content of the universe. Their method does not allow to obtain both the equation of state parameters and the transition redshift in the same framework. The dark energy equation of state parameters are estimated in models with different ansatz on $w(z)$. However the constraint on the transition redshift was obtained using other relation for $d_L(z)$. Let us note that there is no reason to treat the value of z_T estimated in a certain model as a real value in another model.

In this paper we propose the method which allows to calculate both equation of state parameters and the transition redshift in the same model. We estimate the value of z_T through the potential function $V[a(z)]$ of the Hamiltonian system determining the evolution of the universe. In this approach the evolution of the universe is reduced to the motion of a fictitious particle with a unit mass in a one-dimensional potential. The information about the influence of matter content on the dynamics of the universe is derived from the shape of the potential function $V[a(z)]$. Namely the scale factor a accelerates (decelerates) in the interval of z if $V[a(z)]$ is an increasing (decreasing) function of z . Therefore we should expect the potential function has a maximum for the universe with deceleration during its earlier epoch and acceleration at present.

In our previous papers [4, 5] we developed the potential function method to probe the dark energy. The use of the potential $V[a(z)]$ instead of $w_X(z)$ has an advantage because the former suffers less from the smearing effect caused by the double integral which relates $w_X(z)$ and $d_L(z)$ [6]. In these papers we showed that the dynamics of the Friedmann-Robertson-Walker (FRW) model of the universe, filled with non-interacting, non-relativistic matter with equation of state $p_m = w_m(a)\rho_m$ and dark energy with the equation of state $p_X = w_X(a)\rho_X$, can be reduced to the Hamiltonian system

$$\mathcal{H} = \frac{\dot{a}^2}{2} + V(a), \quad (1)$$

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where an overdot denotes the differentiation with respect to the cosmic time t , and the potential

$$V(a) = -\frac{\rho_{\text{eff}} a^2}{6}, \quad \rho_{\text{eff}}(a) = \rho_m(a) + \rho_X(a), \quad (2)$$

where the units $8\pi G = c = 1$ are used and ρ_{eff} is the effective energy density of the mixture of non-interacting fluids which satisfy the conservation equation

$$\dot{\rho}_i = -3\frac{\dot{a}}{a}(\rho_i + p_i). \quad (3)$$

Then ρ_{eff} also satisfies equation (3). On the other hand the potential function $V(a)$ can be reconstructed from the SN Ia data through the Hubble function $H(z)$ which in the spatially flat universe is related to the luminosity distance $d_L(z)$ as

$$H(z) = \left[\frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right]^{-1}. \quad (4)$$

Because the trajectories of system (1) always lie on the zero-energy level (as a consequence of the Hamiltonian constraint) we can rewrite potential (2) to the form

$$V(z) = -\frac{1}{2} \left[(1+z) \frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right]^{-2}. \quad (5)$$

The potential $V(z)$ contains all information necessary to determine the full dynamics of the universe on the phase plane (a, \dot{a}) . Thus both the effective energy density $\rho_{\text{eff}}(a)$ and the equation of state coefficient $w_{\text{eff}}[a(z)] \equiv p_{\text{eff}}/\rho_{\text{eff}}$ can be calculated unambiguously from the potential function $V(a)$

$$\rho_{\text{eff}}(a) = -\frac{6V(a)}{a^2}, \quad w_{\text{eff}}(a) = -\frac{1}{3}[1 + I_V(a)], \quad (6)$$

where $I_V(a) = (d \ln V)/(d \ln a)$ is the elasticity of the potential function V with respect to the scale factor a .

As it is well known the behavior of the potential function in the neighborhood of a maximum can be approximated by a quadratic part of the expansion in Taylor's series. Therefore we have

$$V(a) = V(a_T) + \frac{1}{2} \frac{\partial^2 V}{\partial a^2} \Big|_{a=a_T} (a - a_T)^2 \quad (7)$$

where $V(a_T) = -\dot{a}_T^2/2$ from the Hamiltonian constraint, and

$$\rho_{\text{eff}}(a) = -\frac{6}{a^2} \left[-\frac{\dot{a}_T^2}{2} + \frac{1}{2} \frac{\partial^2 V}{\partial a^2} \Big|_{a=a_T} (a - a_T)^2 \right] \quad (8)$$

or

$$\begin{aligned} \rho_{\text{eff}}(z) = & -3(1+z_T)^4 \frac{\partial^2 V}{\partial z^2} \Big|_{z=z_T} \\ & + 3(1+z)^2 \left[\frac{1}{(1+z_T)^4} \left(\frac{dz}{dt} \Big|_{z=z_T} \right)^2 - (1+z_T)^2 \frac{\partial^2 V}{\partial z^2} \Big|_{z=z_T} \right] \\ & - 6(1+z)(1+z_T)^3 \frac{\partial^2 V}{\partial z^2} \Big|_{z=z_T}. \end{aligned} \quad (9)$$

In the case of transition from deceleration to acceleration one can see from formula (9) that there are three terms which can be interpreted as the cosmological constant (positive), the curvature contribution (negative), and the topological defects with positive energy ($p = -2/3\rho$). Note that as z is close to -1 ($a \gg a_0$) than the last two terms are negligible and only the “cosmological constant” dominates the dynamics of the late epoch of the universe.

The value of the potential function at the maximum is given by

$$V(a_T) = -\frac{1}{2} H_T^2 (1+z_T)^{-2}. \quad (10)$$

The value of z_T and $V(z_T)$ inform us about the moment of transition and the Hubble function at this moment. The estimation of these parameters will be done in the next section.

II. ESTIMATION OF REDSHIFT TRANSITION

In our previous papers the method of reconstruction of the potential function was applied to the analysis of global dynamics of the FRW models in the phase space [4, 5, 7]. The potential function was reconstructed from the SN Ia data which allow us to determine all evolutionary paths of the model for all admissible initial conditions. These results have the qualitative character, but it is also possible to obtain some quantitative attributes of the reconstructed dynamics of a model.

It seems to be important to determine when the switch from the deceleration to the acceleration happens in different cosmological models. Because it is possible to differentiate between cosmological models through the quantitative features of their potential functions, we propose to use it as the test of cosmological models. The maximum of the potential function, interpreted as the time of switch between deceleration and acceleration phases, can be chosen as a one of the criteria (another may be the value of the potential function at z_T). If someone obtains the value of z_T independently from any model (e.g. one can numerically probe it directly from the observational data), it could be compared with values of z_T found in different cosmological models.

We calculate the probability distribution of the transition redshift for the spatially flat FRW model filled with nonrelativistic matter ($\rho_m = \rho_{m,0}a^{-3}$) and dark energy for which the linear parametrization of the equation of state is assumed ($w(z) = w_0 + w_1z$). In our analysis we use the recently released SN Ia data by Riess et al. [3]. We use both the gold subset and gold+silver (full) sample. To estimate the cosmological model parameters w_0 , w_1 , and the transition redshift z_T ($\Omega_{m,0} = 0.3$ and $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ priors are used) we minimize the χ^2 statistics given by the formula

$$\chi^2(\Omega_{m,0}, H_0, w_0, w_1) = \sum_j \frac{[\mu_{t,j}(z_j; \Omega_{m,0}, H_0, w_0, w_1) - \mu_{o,j}]^2}{\sigma_{\mu_{o,j}}^2}, \quad (11)$$

where $\mu_{o,j} = m_{o,j} - M$ is the extinction-corrected distance modulus for SN Ia at redshift z_j , $\sigma_{\mu_{o,j}}$ is the uncertainty in the distance moduli $\mu_{o,j}$ including the dispersion in galaxy redshift due to peculiar velocities, $\mu_{t,j}(z_j; \Omega_{m,0}, H_0, w_0, w_1) = 5 \log d_L(z_j; \Omega_{m,0}, H_0, w_0, w_1) + 25$,

$$\begin{aligned} d_L(z_j; \Omega_{m,0}, H_0, w_0, w_1) &= (1 + z_j) \int_0^{z_j} \frac{dz'}{H(z'; \Omega_{m,0}, H_0, w_0, w_1)} \\ &= (1 + z_j) H_0^{-1} \int_0^{z_j} \frac{dz'}{\sqrt{\Omega_{m,0}(1 + z')^3 + \Omega_{X,0}(1 + z')^{3(1+w_0-w_1)} \exp(3w_1z)}}. \end{aligned} \quad (12)$$

The summation in equation (11) is over all of the observed supernovae. The potential function for the model assumes the form

$$\begin{aligned} V[a(z), \Omega_{m,0}, H_0, w_0, w_1] &= -\frac{1}{2} H^2(a(z), \Omega_{m,0}, H_0, w_0, w_1) a^2(z) = -\frac{1}{2} \left[\frac{d}{dz} \left(\frac{d_L(a(z), \Omega_{m,0}, H_0, w_0, w_1)}{1 + z} \right) \right]^{-2} a^2(z) \\ &= -\frac{1}{2} H_0^2 \left[\Omega_{m,0}(1 + z) + \Omega_{X,0}(1 + z)^{3(1+w_0-w_1)-2} \exp(3w_1z) \right]. \end{aligned} \quad (13)$$

TABLE I: Fitting results of the statistical analysis from distant type Ia supernovae data. (Sample of 186 SN Ia, Riess et al. (2004))

Sample	$\Omega_{m,0}$	$\Omega_{X,0}$	w_0	w_1	χ^2	z_T	$V(z_T)$	Method
gold	0.30	0.70	-1.35	1.52	175.57	0.40	-1.60	best fit
	0.30	0.70	$-1.33^{+0.19}_{-0.19}$	$1.49^{+0.81}_{-0.94}$		$0.38^{+0.10}_{-0.07}$	$-1.58^{+0.05}_{-0.06}$	max(P)
gold + silver	0.30	0.70	-1.41	1.73	229.36	0.38	-1.58	best fit
	0.30	0.70	$-1.39^{+0.17}_{-0.18}$	$1.70^{+0.75}_{-0.85}$		$0.37^{+0.07}_{-0.06}$	$-1.58^{+0.05}_{-0.04}$	max(P)

The best fitted parameters of the model as well as the transition redshift z_T and the maximum of the potential $V_T = V(z_T)$ are presented in Table I.

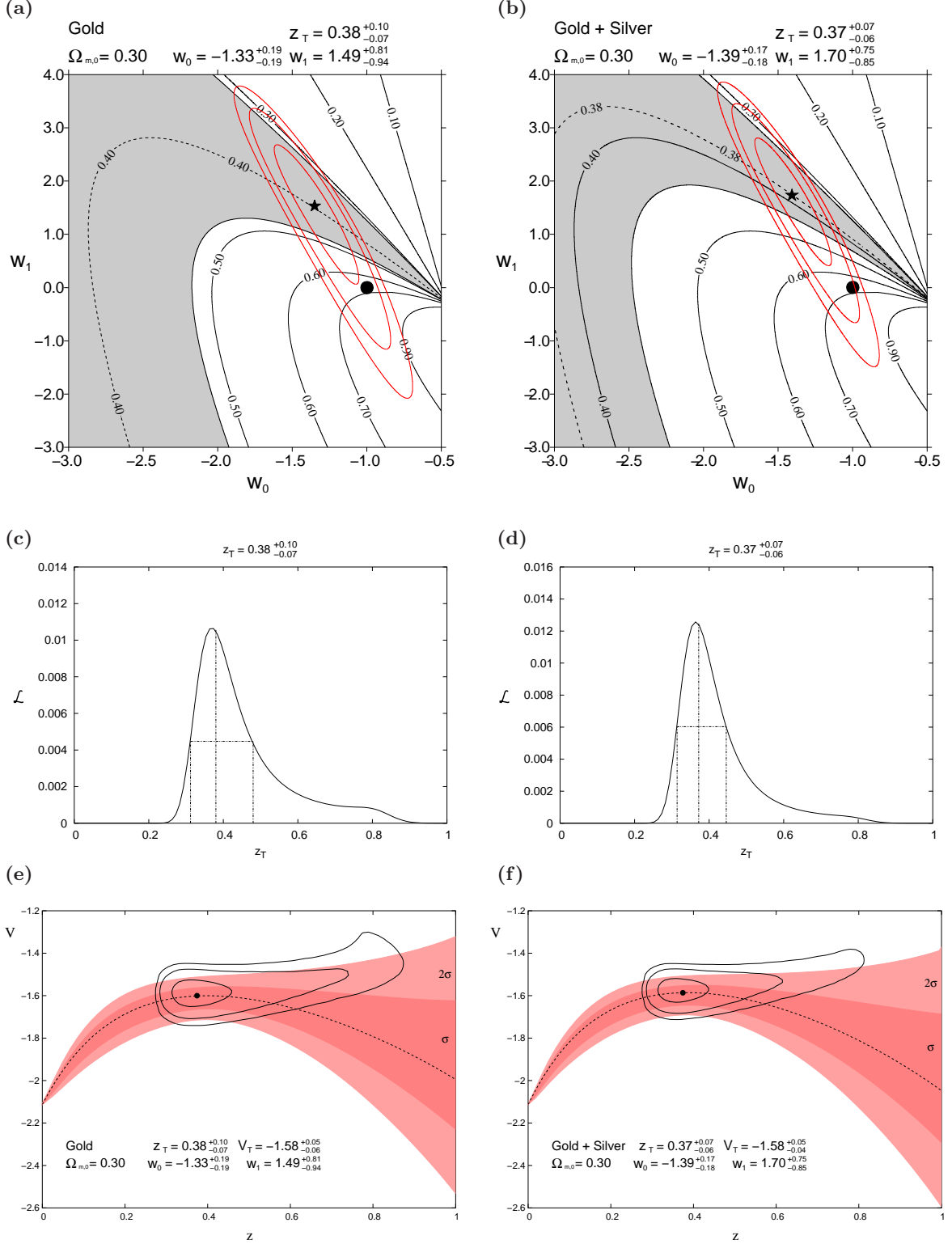


FIG. 1: The left column refers to the gold subset of Riess et al.'s SN Ia sample and the right column to the gold+silver sample. It is assumed that $\Omega_{m,0} = 0.30$ and $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}$. Figures (a) and (b) present the levels of constant transition redshift on the plane (w_0, w_1) as well as the confidence levels (68%, 95% and 99%) for pairs of (w_0, w_1) parameters obtained from the maximum likelihood method. The star denotes the best fit values for $w_0 = -1.35$, $w_1 = 1.52$ (gold subset) and for $w_0 = -1.41$, $w_1 = 1.73$ (gold+silver sample). The dot denotes the cosmological constant model. The shaded region is the 1σ confidence level for z_T from the likelihood method around the best fit value (the dashed line). Figures (c) and (d) present the probability distribution of the transition redshift z_T . Figures (e) and (f) show the confidence levels for the reconstructed potential function $V(z)$ as well as for the maximum of the potential function $(z_T, V(z_T))$.

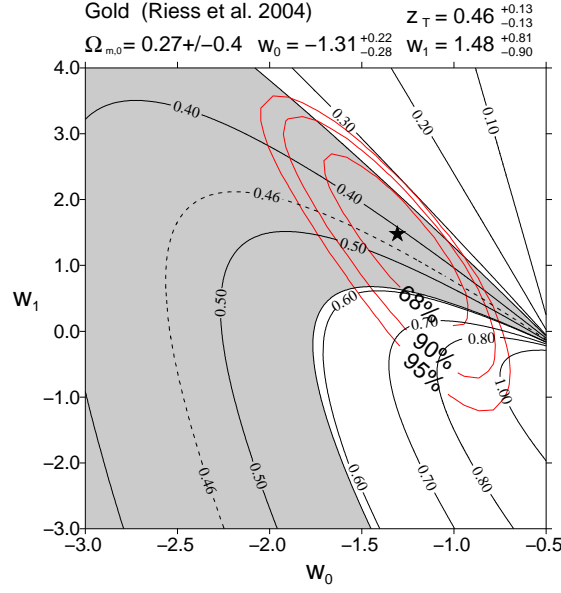


FIG. 2: We draw the levels of constant z_T on the plane (w_0, w_1) . We distinguish $z_T = 0.46$ (the dashed line) with its 1σ confidence level. We also mark with the star the best fit of (w_0, w_1) and the 68%, 95% and 99% confidence levels obtained by Riess et al. [3]. The key observation is that the transition redshift for their (w_0, w_1) parameters is lower than the corresponding value obtained from the Turner-Riess kinematic description [8].

Assuming that the measurement uncertainties are Gaussian we can simply calculate the likelihood function $\mathcal{L}(\Omega_{m,0}, H_0, w_0, w_1)$ from a chi-squared statistics in the following way

$$\mathcal{L}(\Omega_{m,0}, H_0, w_0, w_1) = \frac{1}{N} \exp \left(-\frac{\chi^2(\Omega_{m,0}, H_0, w_0, w_1)}{2} \right),$$

$$N = \iiint \int_{-\infty}^{\infty} \exp \left(-\frac{\chi^2(\Omega_{m,0}, H_0, w_0, w_1)}{2} \right) d\Omega_{m,0} dH_0 dw_0 dw_1, \quad (14)$$

where N is the normalization coefficient given as an integral of the likelihood function over all probability variables. The confidence levels (68%, 95%, 99%) for pairs (w_0, w_1) , obtained for the gold subset and full sample, are drawn in Fig. 1(a) and Fig. 1(b) respectively. We find the one-dimensional probability distribution functions for w_1 and w_2 separately by integrating the likelihood function \mathcal{L} over remaining probability variables and we obtain $w_0 = -1.33^{+0.19}_{-0.18}$, $w_1 = 1.49^{+0.81}_{-0.94}$ for the gold subset and $w_0 = -1.39^{+0.17}_{-0.18}$, $w_1 = 1.70^{+0.75}_{-0.85}$ for the full sample.

We find the probability distribution for the transition redshift z_T by integrating the likelihood function \mathcal{L} along the levels of constant z_T (see Fig. 1(c),(d)). As a result we obtain $z_T = 0.38^{+0.10}_{-0.07}$ (gold subset) and $z_T = 0.37^{+0.07}_{-0.06}$ (gold+silver sample).

We also calculate the probability distribution for the value of the potential $V(z_T)$ by integrating the likelihood function \mathcal{L} along the levels of constant $V(z_T)$ for the best-fitted value of z_T . In this case we have $V(z_T) = -1.58^{+0.05}_{-0.06}$ in units of $10^3 H_0^2$ (gold subset), $V(z_T) = -1.58^{+0.05}_{-0.04}$ in units of $10^3 H_0^2$ (gold+silver sample).

Because of the Hamiltonian constraint we can also calculate the value of the Hubble parameter corresponding to transition redshift z_T

$$H_T = (1 + z_T) \sqrt{-2V(z_T)} \quad (15)$$

and the velocity of the scale factor during the transition epoch

$$\dot{a}_T = \sqrt{-2V(z_T)} \quad (16)$$

and obtain that $H_T \simeq 79.2 > H_0$ and $\dot{a}_T \simeq 56.6 < \dot{a}_0 = H_0$ (gold subset).

The quality of the fitting of the potential function as well as the position of its maximum is illustrated in Fig. 1 (e), (f). It is interesting that the value of $V(z_T)$ is calculated with lower error than z_T . We also see that the errors

decrease in the neighbourhood of the region around the maximum of $V(z_T)$. For the gold+silver sample the errors are lower than for the gold subset.

We estimated the value of z_T in the model with linear form of the equation of state coefficient $w(z)$. The value of transition redshift was also obtained by Riess et al. in the different model. To compare their result in context of the presented framework we look for the transition redshift for their estimated model (model with linear ansatz on $w(z)$). We obtained lower value (marked as a star in Fig. 2) than their corresponding value 0.46 (marked as the dashed line). In our opinion the reason of this difference is that their estimation of z_T was done in different model without any priors on model parameters.

III. CONCLUSIONS

We presented the V-method of reconstruction of the potential function. We applied it to the estimation of the state parameters of the cosmological model with dark energy. We concentrated on the estimation of the position of the maximum of the potential function which can be interpreted as the moment of switch from the deceleration to the acceleration phase during the Universe evolution.

The shape of the potential function is reconstructed from the distant SN Ia data. For our analysis we used the samples of SN Ia prepared by Riess et al.

The Hamiltonian formulation of the dynamics enables us to determine whole dynamics of the cosmological model from the potential function only. It is interesting that this function determines not only the qualitative structure of the phase space [4, 5] but also gives us some quantitative information about the model parameters.

We showed that the V-method is very effective in analysis of cosmological models with dark energy where we can find the moment of transition as the quantitative parameter. The other obtained parameter of state of the system is the velocity of the scale factor (or the Hubble parameter) at the moment of transition. We can treat these two estimated parameters as initial conditions and determine uniquely the evolution path of the model.

The main results of using this method to the cosmological model with dark energy are

1. The method allows to study quantitative and qualitative aspects of dynamics of models.
2. We showed that the state parameters at the moment of transition can be obtained in any models for which the reconstruction can be done.
3. We estimated the value of the transition redshift and the value of Hubble function at the transition moment.

Riess et al. estimated the model parameters assuming the different forms of the equation of state coefficient $w(z)$, but to find the transition redshift z_T they used a certain kinematic model with a linear expansion for $q(z)$. It seems to be no a priori reasons to extrapolate the value of z_T between two different models. The question is to which $w(z)$ model corresponds their value of z_T .

Our methods overcomes this obstacle. To obtain the model parameters and the transition redshift we use the same $d_L(z)$ relation.

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